(8 Pages) **Reg. No. :** ....

Code No. : 6850 **Sub. Code : PMAE 32**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Third Semester

Mathematics

Elective—CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If F is a function of x, y', the Euler's equation is

   (a) $F - y' F_{y'} = C$ (b) $F_{y'} = C$

   (c) $xF_{y'} - F_x = 0$ (d) $xF_{y'} = C$

2. Volume of ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is $f=$

(a) x^2yz (b) $8xyz^2$

(c) $8xyz$ (d) xyz^2

3. $\delta(F_1F_2)=$

(a) $F_1\delta F_2$ (b) $(\delta F_1)F_2$

(c) $F_1\delta F_2+F_2\delta F_1$ (d) $F_1\delta F_2-F_2\delta F_1$

4. A stationary function for an integral functional is one for which the variation of the integral is

(a) positive (b) negative

(c) zero (d) constant

5. $y(x)=1+\lambda\int_0^1(1-3x\xi)y(\xi)\,d\xi$ is a ______________ integral equation.

(a) I kind volterra

(b) II kind volterra

(c) III kind volterra

(d) Fredholm

6. If $G(x,\xi)$ is the Green's function, then $G_2'(\xi) - G'_1(\xi) =$

(a) 0 (b) 1

(c) $\frac{1}{p(\xi)}$ (d) $-\frac{1}{p(\xi)}$

7. If $K(x,\xi)$ is a polynomial in x and ξ then it is a ———————— Kernal.

(a) Continuous (b) Separable

(c) Symmetric (d) None separable

8. The solution of $y(x) = F(x) + \lambda \int_a^b K(x, \xi / y(\xi) d\xi$ can be found by iterative method if $|\lambda|$

(a) $= \frac{1}{M(b-a)}$ (b) $< \frac{1}{M(b-a)}$

(c) $> \frac{1}{M(b-a)}$ (d) $= 0$

9. The characteristic functions $y_m(x)$ and $y_n(x)$ of the homogeneous Fredholm equation $y(x) = \lambda \int_a^b K(x,\xi)\, y(\xi)\, d\xi$ are

(a) real

(b) differ by a constant

(c) orthogonal

(d) equal

10. The characteristic numbers of a Fredholm equation with a non-symmetric Kernal are

(a) real

(b) complex

(c) equal

(d) proprotional

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Prove that the shortest distance between two points in a plane is a straight line.

Or

(b) Determine the stationary function of $I = \int_0^1 y'^2 f(x)\,dx$, $y(0) = 0$, $y(1) = 1$ where

$$f(x) = \begin{cases} -1, & 0 \le x < \frac{1}{4} \\ 1, & \frac{1}{4} < x \le 1. \end{cases}$$

12. (a) (i) Find ΔF for $F = F(x, y, u, v, u_x, u_y, v_x, v_y)$.

(ii) If $I = \int_0^1 (x^2 - y^2 + y'^2)\,dx$, find ΔI and δI when $dy = \in x^2$.

Or

(b) Solve $I = \int_{x_1}^{x_2} \sqrt{1 + y'^2} \, dx$, $y(x_1) = y_1$, $y(x_2) = g(x_2)$ where $g(x) = mx + b$, m, b are constants

13. (a) State the properties of Green's function.

Or

(b) Transform $y'' + xy = 1$, $y(0) = 0$, $y(l) = 1$ into an integral equation.

14. (a) Obtain a n approximate solution of the integral equation

$$y(x) = \int_0^1 \sin(x\xi)\, y(\xi)\, d\xi + x^2 .$$

Or

(b) Obtain the volterra equation of the second kind.

15. (a) Solve $y(x) = 1 + \lambda \int_0^1 (1 - 3x\xi)\, y(\xi)\, d\xi$ by iterative methods.

Or

(b) Show that if $F(x) = \int_a^b K(x, \xi)\, y(\xi)\, d\xi$ possesses a continuous solution, then it is of the form $y(x) = \sum_n \lambda_n f_n \phi_n(x) + \phi(x)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Of all rectangular parallelepipeds which have sides parallel to coordinates planes and which are inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, determine the dimensions of that one which has the largest possible volumes.

Or

(b) Find the minimal surface of revolution passing through two points.

17. (a) Obtain the partial differential equation satisfied by the equation of a minimal surface.

Or

(b) Illustrate the Dirichlet problem.

18. (a) Show that $y(x)=\int_a^b G(x,\xi)\phi(\xi)\,d\xi$ is an integral formulation of the differential equation $Ly+\phi(x)=0$ with homogeneous boundary conditions $\alpha y+\beta y'=0$, where $L=p\frac{d^2}{dx^2}+\frac{dp}{dx}\frac{d}{dx}+q$.

Or

(b) Transform the problem

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (\lambda x^2 - 1)\, y = 0\,, y(0) = 0\,, \quad y(1) = 0$$

into an integral equation.

19. (a) Suppose that a string is rotating uniformly about the $x-$axis with angular velocity w. Show that the influence function is the Green's function of the problem.

Or

(b) Explain the procedure to solve Fredhom equations of second kind with separable Kernals.

20. (a) Determine the characteristic values and the corresponding characteristic functions of the integral equation

$$y(x) = \lambda \int_0^1 (1 - 3x\xi)\, y(\xi)\, d\xi + F(x).$$

Or

(b) Explain an iterative method for solving a volterra equation.